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Noncommutative cosmological models coupled to a perfect fluid and a cosmological constant

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ABSTRACT: In this work we carry out a noncommutative analysis of several Friedmann-Robert-Walker models, coupled to different types of perfect fluids and in the presence of a cosmological constant. The classical field equations are modified, by the introduction of a shift operator, in order to introduce noncommutativity in these models. We notice that the noncommutative versions of these models show several relevant differences with respect to the correspondent commutative ones.

KEYWORDS: Classical Theories of Gravity, Integrable Field Theories, Non-Commutative Geometry

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1 Introduction

In current theoretical physics there is a relevant number of theoretical investigations that lead us to believe that at the Big-Bang first moments, the geometry was not commutative and the dominating physics at that time was ruled by the laws of noncommutative (NC) geometry. Therefore, the idea is that the physics of the early moments can be constructed based on these concepts.

The first published steps through this knowledge were given by Snyder [1] who believes that NC principles could make the quantum field theory infinities disappear. However, it was not accomplished [2] and Snyder's ideas were put to sleep for a long time. The main modern motivations that rekindle the investigation about NC field theories came from string theory and quantum gravity [3].

In the context of quantum mechanics for example, R. Banerjee [4] discussed how NC structures appear in planar quantum mechanics providing a useful way for obtaining them. The analysis was based on the NC algebra in planar quantum mechanics that was originated from 't Hooft's analysis on dissipation and quantization [5].

It is opportune to mention here that this noncommutativity in the context of string theory mentioned above could be eliminated constructing a mechanical system which reproduces the string classical dynamics [6]. NC field theories have been studied intensively in many branches of physics [7]-[14].

In a very interesting paper, a parallel investigation was developed by Duval and Horváthy [15], where it was obtained the anomalous commutation relations for the coordinates obtained through the “Peierls substitution” [16]. From first principles, without using such unphysical limit, the authors introduced NC (quantum) mechanics starting with group theory and applied it to condensed matter physics, e.g., the Hall effect. The respective Lagrangian approach was discussed in detail in subsequent papers [17]. Dunne, Jackiw and Trugenberger [18] justify the Peierls rule by considering the $m \rightarrow 0$ limit, reducing the classical phase space from four to two dimensions, parameterized by NC coordinates X and Y , whereas the potential becomes an effective Hamiltonian.

In [19] the authors analyzed the IR and UV divergences and verified that Planck’s constant enters via loop expansion. Here, differently, we make a non-perturbative approach and we will see that Planck’s constant enters naturally in the theory via Moyal-Weyl product.

A general algebra α -deformation of classical observables that introduces a general NC quantum mechanics was constructed in [20]. This α -deformation is equivalent to some general transformation for the usual quantum phase space variables. In other words, the authors discuss the passage from classical mechanics to quantum mechanics. Then to NC quantum mechanics, which allows to obtain the associated NC classical mechanics. This is possible since quantum mechanics is naturally interpreted as a NC (matrix) symplectic geometry [21].

In [22], the author constructed an extension of the well known Doplicher-Fredenhagen-Roberts NC algebra introducing the formalism which is now called in the literature as the Doplicher-Fredenhagen-Roberts-Amorim algebra. In this formalism the NC parameter (θ) is an ordinary coordinate of the spacetime and therefore it has a canonical conjugate momentum (π). An extended Hilbert space was constructed together with all the ingredients of a new NC quantum mechanics. But notice that both preserves the underlying NC relation $[x^\mu, x^\nu] = i\theta^{\mu\nu}$. For details, the interested reader can consult [23].

Back to our main subject here, in few words we can say that one way to obtain NC versions for field theories one have to replace the usual product of fields into the action by the Moyal-Weyl product, defined as

$$\phi_1(x) \star \phi_2(x) = \exp\left(\frac{i}{2}\theta^{\mu\nu}\partial_\mu^x\partial_\nu^y\right)\phi_1(x)\phi_2(y)|_{x=y}, \quad (1.1)$$

where $\theta^{\mu\nu}$ is a real and antisymmetric constant matrix. As a consequence, NC theories are highly nonlocal. We also note a basic NC property that the Moyal-Weyl product of two fields inside the action is the same as the usual product, considering that we discard boundary terms. Thus, the noncommutativity affects just the vertices.

Some years back, [24] three of us have proposed a new formalism to generalize the quantization by deformation introduced in [20] in order to explore, with a new insight,

how the NC geometry can be introduced into a (commutative) field theory. To accomplish this, a systematic way to introduce NC geometry into commutative systems, based on Faddeev-Jackiw symplectic formalism and Moyal-Weyl product, was presented [25].

One important arena where NC ideas may play an important role is cosmology. If superstrings is the correct theory to unify all the interactions in nature, it must have played the dominant role at very early stages of our Universe. At that time, all the canonical variables and corresponding momenta describing our Universe should have followed a NC algebra. Inspired by these ideas some researchers have considered such NC models in quantum cosmology [26–28]. It is also possible that some residual NC contribution may have survived in later stages of our Universe. Based on these ideas some researchers have proposed some NC models in classical cosmology in order to explain some intriguing results observed by WMAP. Such as a running spectral index of the scalar fluctuations and an anomalously low quadrupole of CMB angular power spectrum. Among such proposals we may mention the following ones [29–33]. Another relevant application of the NC ideas in classical cosmology is the attempt to explain the present accelerated expansion of our Universe [34–36].

We have organized this paper as follows. In section 2, we introduce the generalized quantization by deformation formalism assuming a generic classical symplectic structure. We will construct a new star product which appears at first sight to suffer from the non-associative property disease. We will demonstrate that we can recover this crucial property for the star product. In section 3, we construct the NC versions of several Friedmann-Robertson-Walker cosmological models. These models may have positive, negative or zero spatial sections curvatures, they are coupled to different types of perfect fluids and they may have a positive, negative or zero cosmological constant. We have used the Schutz variational formalism in order to write the Hamiltonian of the models. In section 4, we perform a complete study of the evolution of the universes described by each model. We solve the dynamical equations for the scale factor. Based on our results, we conclude how the NC parameter modifies the evolution of the universe in different models. Also in this section, we present some special cases where our conclusions can be clearly verified. In section 5, based on our previous results, we write a function ($\tilde{\Lambda}$) that depends on three parameters that generalizes the cosmological constant. We discuss the possible scenarios for the Universe evolution predicted by $\tilde{\Lambda}$, depending on the value of α . The final considerations and the conclusions are depicted in the last section.

2 The NC Generalized Symplectic Formalism

The quantization by deformation [37] consists in the substitution of the canonical quantization process by the algebra \mathcal{A}_\hbar of quantum observables generated by the same classical one obeying Moyal-Weyl product, *i.e.*, the canonical quantization

$$\{h, g\}_{PB} = \frac{\partial h}{\partial \zeta_a} \omega_{ab} \frac{\partial g}{\partial \zeta_b} \longrightarrow \frac{1}{i\hbar} [\mathcal{O}_h, \mathcal{O}_g] , \quad (2.1)$$

with $\zeta = (q_i, p_i)$, is replaced by the \hbar -star deformation of \mathcal{A}_0 , given by

$$\{h, g\}_h = h \star_h g - g \star_h h , \quad (2.2)$$

where

$$(h \star_h g)(\zeta) = \exp\left\{\frac{i}{2}\hbar\omega_{ab}\partial_{(\zeta_1)}^a\partial_{(\zeta_2)}^b\right\}h(\zeta_1)g(\zeta_2)|_{\zeta_1=\zeta_2=\zeta} , \quad (2.3)$$

with $a, b = 1, 2, \dots, 2N$ and with the following classical symplectic structure

$$\omega_{ab} = \begin{pmatrix} 0 & \delta_{ij} \\ -\delta_{ji} & 0 \end{pmatrix} \text{ with } i, j = 1, 2, \dots, N , \quad (2.4)$$

which satisfies the relation

$$\omega^{ab}\omega_{bc} = \delta_c^a . \quad (2.5)$$

In the next section we will describe the conceptual basis that support our method and our cherished results. However, we think that it is crucial for the reader to understand mathematically the underlying equations used in this work. We will show that the crucial property of associativity is not lost in our formalism.

2.1 Deformation quantization

Notice that we have a subtle conceptual analogy between both star products, i.e., the one defined by the Moyal-Weyl product in Eq. (1.1) and the Σ -star deformation product defined in Eq. (2.3). In general the star products so defined are associative if the parameter is constant. If we do not have associativity, the product so defined is useless in physics. We will talk with more detail about this from now on.

Kontsevich proved in [38] that any finite-dimensional Poisson manifold can be canonically quantized (deformation quantization). We will write now a quite brief introduction to deformation quantization following [38].

Let us define an algebra $A = \Gamma(X, \mathcal{O}_X)$ over \mathcal{R} of smooth functions on a finite-dimensions C^∞ -manifold X . We construct a star product on A defined as being an associative $R[\hbar]$ -linear product on $A[\hbar]$. This star product between f and g ($f, g \in A \subset A[\hbar]$) can be written as

$$(f, g) \rightarrow f \star g = fg + \hbar B_1(f, g) + \hbar^2 B_2(f, g) + \dots \in A[\hbar] \quad (2.6)$$

where \hbar is a constant parameter and B_i are bidifferential operators. A bidifferential operator can be understood as bilinear maps which are differential operator [38]. And the product of arbitrary elements of $A[\hbar]$ can be defined following the framework written in (2.6) and it is defined by the condition of linearity over $\mathcal{R}[\hbar]$

$$\left(\sum_{n \geq 0} f_n \hbar^n\right) \star \left(\sum_{n \geq 0} g_n \hbar^n\right) = \sum_{k, l \geq 0} f_k g_l \hbar^{k+l} + \sum_{k, l \geq 0, m \geq l} B_M(f_k, g_l) \hbar^{k+l+m} .$$

For more details the interested reader can look in [38].

The simplest of a deformation quantization is the Moyal-Weyl product for the Poisson structure on \mathcal{R}^d with constant coefficients,

$$\begin{aligned} f \star g &= f g + \hbar \sum_{i,j} \alpha^{ij} \partial_i(f) \partial_j(g) + \frac{\hbar^2}{2} \sum_{i,j,k,l} \alpha^{ij} \alpha^{kl} \partial_i \partial_k(f) \partial_j \partial_l(g) + \dots \\ &= \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \sum_{i_1, \dots, i_n, j_1, \dots, j_n} \prod_{k=1}^n \alpha^{i_k j_k} \left(\prod_{k=1}^n \partial_{i_k} \right) (f) \cdot \left(\prod_{k=1}^n \partial_{j_k} \right) (g) \end{aligned} \quad (2.7)$$

where $\alpha^{ij} = -\alpha^{ji}$.

Let $\alpha = \sum_{i,j} \alpha^{ij} \partial_i \wedge \partial_j$ be a Poisson bracket with variable coefficients in an open domain of \mathcal{R}^d [38]. Namely, α^{ij} is not a constant but a function of coordinates. Then the following star product gives an associative product modulo $O(\hbar^3)$ [38],

$$\begin{aligned} f \star g &= f g + \hbar \sum_{i,j} \alpha^{ij} \partial_i(f) \partial_j(g) + \frac{\hbar^2}{2} \sum_{i,j,k,l} \alpha^{ij} \alpha^{kl} \partial_i \partial_k(f) \partial_j \partial_l(g) \\ &+ \frac{\hbar^3}{3} \sum_{i,j,k,l} \alpha^{ij} \partial_j(\alpha^{kl}) \left(\partial_i \partial_k(f) \partial_l(g) - \partial_k(f) \partial_i \partial_l(g) \right) + O(\hbar^3). \end{aligned} \quad (2.8)$$

For us to demonstrate the associativity up to the second order means that for any three functions f, g and h we have that

$$(f \star g) \star h = f \star (g \star h) \quad (2.9)$$

We can see clearly from (2.8) that the fact of α^{ij} being a non-constant parameter brings a different \hbar^3 order for the Moyal-Weyl product. A simple comparison between Eqs. (2.7) and (2.8) can make us see that if we can not write a star product like the one described in (2.8) so we do not have associativity. We will turn back to this issue in a few moments.

The quantization by deformation can be generalized assuming a generic classical symplectic structure Σ^{ab} . In this way the internal law will be characterized by \hbar and by another deformation parameter (or more). As a consequence, the Σ -star deformation of the algebra becomes

$$(h \star_{\hbar \Sigma} g)(\zeta) = \exp\left\{ \frac{i}{2} \hbar \Sigma_{ab} \partial_{(\zeta_1)}^a \partial_{(\zeta_2)}^b \right\} h(\zeta_1) g(\zeta_2) |_{\zeta_1 = \zeta_2 = \zeta}, \quad (2.10)$$

with $a, b = 1, 2, \dots, 2N$.

This new star-product generalizes the algebra among the symplectic variables in the following way

$$\{h, g\}_{\hbar \Sigma} = i \hbar \Sigma_{ab} \partial_{(\zeta_1)}^a \partial_{(\zeta_2)}^b h(\zeta_1) g(\zeta_2) |_{\zeta_1 = \zeta_2 = \zeta}. \quad (2.11)$$

Notice that the new star product in (2.10) is defined in an analogous way as the Moyal-Weyl product. However, in Eq. (2.11) we see that the parameter Σ_{ab} is not constant. Hence, we can realize that from Eqs. (2.10) and (2.11) the associativity property was lost in Eq. (2.10).

However, if we consider only terms up to \hbar^2 and due to the smallness of \hbar we can recover the associativity property of Eq. (2.10). So, the general expression given in Eq.

(2.10) hides the fact that the expansion of the exponential is physically valid only for terms proportional to $1, \hbar$ and \hbar^2 and therefore it is not completely correct. Our objective in Eq. (2.10) was to write the Σ -star product in a compact way. But we have to make this observation.

On the other hand, we will see soon that Eq. (2.10) is not the starting point of our procedure. The main point is the generalized Dirac quantization defined in the next section.

The motivation to construct a Σ -star deformed product like the one in Eq. (2.10) is to show that we can have in physics an alternative and at the same time associative star product with variable coefficients without losing the associativity property as we showed above, i.e., where the expansion stops in the second order of \hbar .

2.2 The NC approach

In [20, 21], the authors proposed a quantization process to transform the NC classical mechanics into the NC quantum mechanics, through generalized Dirac quantization,

$$\{h, g\}_\Sigma = \frac{\partial h}{\partial \zeta_a} \Sigma_{ab} \frac{\partial g}{\partial \zeta_b} \longrightarrow \frac{1}{i\hbar} [\mathcal{O}_h, \mathcal{O}_g]_\Sigma . \quad (2.12)$$

The relation above can also be obtained through a particular transformation onto the usual classical phase space, namely,

$$\zeta'_a = T_{ab} \zeta^b , \quad (2.13)$$

where the transformation matrix is

$$T = \begin{pmatrix} \delta_{ij} & -\frac{1}{12} \theta_{ij} \\ \frac{1}{12} \beta_{ij} & \delta_{ij} \end{pmatrix} , \quad (2.14)$$

with θ_{ij} and β_{ij} being antisymmetric matrices. As a consequence, the original Hamiltonian becomes

$$\mathcal{H}(\zeta_a) \longrightarrow \mathcal{H}(\zeta'_a) . \quad (2.15)$$

The corresponding symplectic structure is

$$\Sigma_{ab} = \begin{pmatrix} \theta_{ij} & \delta_{ij} + \sigma_{ij} \\ -\delta_{ij} - \sigma_{ij} & \beta_{ij} \end{pmatrix} , \quad (2.16)$$

$\sigma_{ij} = -\frac{1}{18} [\theta_{ik} \beta_{kj} + \beta_{ik} \theta_{kj}]$. Due to this, the commutator relations look like

$$\begin{aligned} [q'_i, q'_j] &= i\hbar \theta_{ij} , \\ [q'_i, p'_j] &= i\hbar (\delta_{ij} + \sigma_{ij}) , \\ [p'_i, p'_j] &= i\hbar \beta_{ij} . \end{aligned} \quad (2.17)$$

At this point, it is important to notice that a Lagrangian formulation was not given so far. Now, we propose a new systematic way to obtain a NC Lagrangian description for a commutative system. In order to achieve our objective, the symplectic structure Σ_{ab} must firstly be fixed and after that, the inverse of Σ_{ab} must be computed. As a consequence, an interesting problem arise: if there are some constants (*Casimir invariants*) in the system,

the symplectic structure has a zero-mode, given by the gradient of these *Casimir invariants*. Hence, it is not possible to compute the inverse of Σ_{ab} . However, in Ref. [39] this kind of problem was solved. On the other hand, if Σ_{ab} is nonsingular, its inverse can be obtained solving the next relation

$$\int \Sigma_{ab}(x, y) \Sigma^{bc}(y, z) dy = \delta_a^c \delta(x - z) , \quad (2.18)$$

which generates a set of differential equations since Σ^{ab} is an unknown two-form symplectic tensor obtained from the following first-order Lagrangian

$$\mathcal{L} = A_{\zeta'_a} \dot{\zeta}'^a - V(\zeta'_a) , \quad (2.19)$$

as being

$$\Sigma^{ab}(x, y) = \frac{\delta A_{\zeta'_a}(x)}{\delta \zeta'_b(y)} - \frac{\delta A_{\zeta'_b}(x)}{\delta \zeta'_a(y)} . \quad (2.20)$$

Due to this, the one-form symplectic tensor, $A_{\zeta'_a}(x)$, can be computed and subsequently, the Lagrangian description, Eq. (2.19), is obtained also.

In order to compute $A_{\zeta'_a}(x)$, the Eq. (2.18) and Eq. (2.20) will be used, which generates the following set of differential equations

$$\begin{aligned} \theta_{ij} B_{jk}(x, y) + (\delta_{ij} + \sigma_{ij}) A_{jk}(x, y) &= \delta_{ik} \delta(x - y) , \\ A_{jk}(x, y) \theta_{ji} + (\delta_{ij} + \sigma_{ij}) C_{jk}(x, y) &= 0 , \\ -(\delta_{ij} + \sigma_{ij}) B_{jk}(x, y) + \beta_{ij} A_{jk}(x, y) &= 0 , \\ A_{kj}(x, y) (\delta_{ji} + \sigma_{ji}) + \beta_{ij} C_{jk}(x, y) &= \delta_{ik} \delta(x - y) , \end{aligned} \quad (2.21)$$

where

$$\begin{aligned} B_{jk}(x, y) &= \left(\frac{\delta A_{q'_j}(x)}{\delta q'_k(y)} - \frac{\delta A_{q'_k}(x)}{\delta q'_j(y)} \right) , \\ A_{jk}(x, y) &= \left(\frac{\delta A_{p'_j}(x)}{\delta q'_k(y)} - \frac{\delta A_{q'_k}(x)}{\delta p'_j(y)} \right) , \\ C_{jk}(x, y) &= \left(\frac{\delta A_{p'_j}(x)}{\delta p'_k(y)} - \frac{\delta A_{p'_k}(x)}{\delta p'_j(y)} \right) . \end{aligned} \quad (2.22)$$

From the set of differential equations in Eq. (2.21), and the equations above, Eq. (2.22), we compute the quantities $A_{\zeta'_a}(x)$.

As a consequence, the first-order Lagrangian can be written as

$$\mathcal{L} = A_{\zeta'_a} \dot{\zeta}'^a - V(\zeta'_a) . \quad (2.23)$$

Notice that, despite (2.19) and (2.23) have the same form, in (2.23) the $A_{\zeta'_a}$ are completely computed through the solution of the system (2.21). In both we have a NC version of the theory as a consequence of the deformation in (2.14) and its corresponding symplectic structure in (2.16).

3 The noncommutative Friedmann-Robertson-Walker models

Friedmann-Robertson-Walker (FRW) cosmological models are characterized by the scale factor $a(t)$ and have the following line element,

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (3.1)$$

where $d\Omega^2$ is the line element of the two-dimensional sphere with unitary radius, $N(t)$ is the lapse function and k gives the type of constant curvature of the spatial sections. The curvature is positive for $k = 1$, negative for $k = -1$ and zero for $k = 0$. Here, we are using the natural unit system, where $c = 8\pi G = 1$. We assume that the matter content of the model is represented by a perfect fluid with four-velocity $U^\mu = \delta_0^\mu$ in the co-moving coordinate system used, plus a cosmological constant (Λ) which can be either positive, negative or zero. The total energy-momentum tensor is given by,

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu - pg_{\mu\nu} - \Lambda g_{\mu\nu}, \quad (3.2)$$

where ρ and p are the energy density and pressure of the fluid, respectively. Here, we assume that $p = \alpha\rho$, which is the equation of state for a perfect fluid.

Einstein's equations for the metric (3.1) and the energy momentum tensor (3.2) are equivalent to the Hamilton equations generated by the following super-Hamiltonian constraint [40],

$$\mathcal{H} = -\frac{P_a^2}{12a} - 3ka + \Lambda a^3 + P_T a^{-3\alpha}, \quad (3.3)$$

where P_a and P_T are the momenta canonically conjugated to a and T respectively, the latter being the canonical variable associated to the fluid. This super-Hamiltonian was derived using the Schutz variational formalism [41, 42]. The starting point to derive the NC version of the above cosmological models is the super-Hamiltonian constraint (3.3).

In order to obtain a NC version for the FRW model, we apply the procedure described in the previous section. Initially, we have to write the zeroth-iterative Lagrangian of the system, which can be done directly from the super-Hamiltonian in (3.3),

$$\mathcal{L}^{(0)} = P_a \dot{a} + P_T \dot{T} - V(a, p_a, T, P_T), \quad (3.4)$$

where $V = N\Omega$ is the symplectic potential and

$$\Omega = \frac{-P_a^2}{12a} - 3ka + \Lambda a^3 + P_T a^{-3\alpha}. \quad (3.5)$$

Notice that the system is treated classically via Symplectic Formalism. From now on we will follow the steps described in the last section.

In the Lagrangian (3.4), the symplectic variables are identified easily as

$$\xi^i = (a, P_a, T, P_T, N),$$

and the corresponding zeroth-iterative one-form canonical momenta

$$\begin{aligned} A_a^{(0)} &= P_a & A_{P_a}^{(0)} &= 0 & A_T^{(0)} &= P_T \\ A_{P_T}^{(0)} &= 0 & A_N^{(0)} &= 0 \end{aligned} \quad . \quad (3.6)$$

Calculating the one-form canonical momenta using the symplectic matrix definition,

$$f_{\xi^i \xi^j} = \frac{\partial A_{\xi^j}}{\partial \xi^i} - \frac{\partial A_{\xi^i}}{\partial \xi^j}, \quad (3.7)$$

we obtain directly the zeroth-iteration symplectic matrix, given by

$$f^{(0)} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (3.8)$$

However, this matrix is singular, which assumes the existence of constraints in the system, and it has the following zero-mode

$$\nu = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (3.9)$$

Multiplying this zero-mode by the symplectic potential gradient we have that,

$$\sum_{i=1}^4 \nu_i \frac{\partial V}{\partial \xi^i} = \Omega, \quad (3.10)$$

where Ω is a constraint. This constraint can be introduced into the kinetic sector of the zeroth-iterative Lagrangian $\mathcal{L}^{(0)}$, through the Lagrangian multiplier β . In this way, the first-iterative Lagrangian can be written as

$$\mathcal{L}^{(1)} = P_a \dot{a} + P_T \dot{T} + \Omega \dot{\beta} - N \Omega, \quad (3.11)$$

and the new set of symplectic variables is $\xi^i = (a, P_a, T, P_T, N, \beta)$. The corresponding first-iterative one-form canonical momenta $A_{\xi^i}(\xi^j)^{(1)}$ are given by

$$\begin{aligned} A_a^{(1)} &= P_a & A_{P_a}^{(1)} &= 0 & A_T^{(1)} &= P_T \\ A_{P_T}^{(1)} &= 0 & A_N^{(1)} &= 0 & A_\beta^{(1)} &= \Omega. \end{aligned} \quad (3.12)$$

Thus, from the relation (3.7) we obtain the first-iterative symplectic matrix

$$f^{(1)} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & \frac{\partial \Omega}{\partial a} \\ 1 & 0 & 0 & 0 & 0 & \frac{\partial \Omega}{\partial P_a} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{\partial \Omega}{\partial P_T} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\partial \Omega}{\partial a} & -\frac{\partial \Omega}{\partial P_a} & 0 & -\frac{\partial \Omega}{\partial P_T} & 0 & 0 \end{pmatrix}. \quad (3.13)$$

However, this matrix is also singular. If we multiply its zero-mode by the gradient of the symplectic potential we will find the same constraint obtained previously,

$$\sum_{i=1}^4 \mu_i \frac{\partial V}{\partial \xi^i} = \Omega. \quad (3.14)$$

This result leads us to conclude that the system has a gauge symmetry. In accordance with the symplectic method, this symmetry must be introduced into the Lagrangian in Eq. (3.11) through the Lagrange multiplier Σ . So, the second-iterative Lagrangian can be written as

$$\mathcal{L}^{(2)} = P_a \dot{a} + P_T \dot{T} + \Sigma \dot{\eta} - N \Omega, \quad (3.15)$$

where $\Sigma = N - 1$, i.e., the lapse function is equal to one, which is equivalent to the choice of a physical time, and the new set of symplectic variables is $\xi^i = (a, P_a, T, P_T, N, \eta)$. Using the symplectic matrix definition in Eq. (3.7) again, we have a non-singular second-iterative symplectic matrix. After a straightforward calculation, we obtain the inverse of the symplectic matrix,

$$(f^{(2)})^{-1} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \quad (3.16)$$

It is important to remember that the elements of this matrix corresponds to the Poisson brackets among the symplectic variables, $(f^{-1})^{ij} = \{\xi^i, \xi^j\}$. The standout moment of the NC introduction through the method described in the last section lies in the assumption that the following relations for the brackets Poisson are,

$$\begin{aligned} \{a, T\} &= \theta \\ \{P_a, P_T\} &= \beta. \end{aligned} \quad (3.17)$$

These brackets are justified by the fact that T and its conjugated momentum P_T can be defined in terms of specific entropy and the potential ϵ [40], which can be written as a function of energy and the position coordinate. Furthermore, in NC classical mechanics the brackets among the coordinates can be nontrivial.

With all these values in mind, using the inverse of the symplectic matrix, including the NC brackets in Eq. (3.17), we can determine the symplectic matrix directly

$$f = \frac{1}{\beta\theta - 1} \begin{bmatrix} 0 & 1 & -\beta & 0 & 0 & 0 \\ -1 & 0 & 0 & -\theta & 0 & 0 \\ \beta & 0 & 0 & 1 & 0 & 0 \\ 0 & \theta & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (\beta\theta - 1) \\ 0 & 0 & 0 & 0 & (1 - \beta\theta) & 0 \end{bmatrix}, \quad (3.18)$$

where $\beta\theta - 1 \neq 0$, is a constraint. To proceed with the method, we use the symplectic matrix elements (3.18) and the relations in Eq. (3.7). The result is a set of partial differential equations,

$$\begin{aligned}
\frac{\partial A_{P_a}}{\partial a} - \frac{\partial A_a}{\partial P_a} &= \frac{1}{\beta\theta - 1} \\
\frac{\partial A_T}{\partial a} - \frac{\partial A_a}{\partial T} &= \frac{-\beta}{\beta\theta - 1} \\
\frac{\partial A_{P_T}}{\partial P_a} - \frac{\partial A_{P_a}}{\partial P_T} &= \frac{-\theta}{\beta\theta - 1} \\
\frac{\partial A_{P_T}}{\partial T} - \frac{\partial A_T}{\partial P_T} &= \frac{1}{\beta\theta - 1} \\
\frac{\partial A_\eta}{\partial N} - \frac{\partial A_N}{\partial \eta} &= 1.
\end{aligned} \tag{3.19}$$

The system above have one possible and convenient solution,

$$\begin{aligned}
A_a &= \frac{1}{1 - \beta\theta}(P_a - \beta T), & A_{P_a} &= \frac{\theta}{\beta\theta - 1}P_T \\
A_T &= \frac{1}{1 - \beta\theta}P_T, & A_{P_T} &= 0 \\
A_\eta &= \Sigma & A_N &= 0.
\end{aligned} \tag{3.20}$$

Therefore, from the one-form canonical momenta above the new NC first-order Lagrangian can be computed directly. However, we also have to consider that the model remains second-order in velocities. Consequently, we will use the canonical momenta given by

$$\begin{aligned}
A_a &= \frac{1}{1 - \beta\theta}(P_a - \beta T) \\
A_T &= \frac{1}{1 - \beta\theta}P_T \\
A_\eta &= \Sigma.
\end{aligned} \tag{3.21}$$

The respective NC first-order Lagrangian is,

$$\mathcal{L} = \frac{1}{1 - \beta\theta}(P_a - \beta T)\dot{a} + \frac{1}{1 - \beta\theta}P_T\dot{T} + \Sigma\dot{\eta} - V(\xi), \tag{3.22}$$

where the symplectic potential is written as

$$V(\xi) = \frac{-P_a^2}{12a} - 3ka + \Lambda a^3 + P_T a^{-3\alpha}. \tag{3.23}$$

In order to obtain a commutative first-order Lagrangian, we propose a coordinate transformation in the classical phase space, analogous to the shift-operator $\hat{x}_i = X_i + \frac{1}{2}\theta_{ij}\hat{p}^j$ used in NC Quantum Mechanics (NCQM), given by

$$\tilde{P}_a = \frac{P_a - \beta T}{1 - \beta\theta}, \quad \tilde{P}_T = \frac{P_T}{1 - \beta\theta}. \tag{3.24}$$

Applying the transformation above in (3.22), we obtain the new commutative first-order Lagrangian,

$$\tilde{\mathcal{L}} = \tilde{P}_a \dot{a} + \tilde{P}_T \dot{T} + \Sigma \dot{\eta} - V(\xi) , \quad (3.25)$$

where the symplectic potential

$$V(\xi) = \frac{-P_a^2}{12a} - 3ka + \Lambda a^3 + P_T a^{-3\alpha} \quad (3.26)$$

is written in NC coordinates that satisfy the usual Poisson brackets given in Eq. (3.17). It is important to note that despite the variables in the Lagrangian Eq. (3.25) are commutative, there are NC contributions into the symplectic potential. Finally, the modified super-Hamiltonian of the system is identified as being the symplectic potential, so it can be written as

$$H = -\frac{(\tilde{P}_a + \beta T)^2}{12a} - 3ka + \Lambda a^3 + \tilde{P}_T a^{-3\alpha}. \quad (3.27)$$

Notice that when $\theta = \beta = 0$ from (3.17) and (3.24), we recover, from (3.27), the Hamiltonian in (3.3).

4 Classical behavior of the NC cosmological models

In order to investigate the contributions coming from the noncommutativity between the canonical variables and momenta in the classical FRW cosmological models, we derive the dynamical equations by computing the Hamilton's equations for the total Hamiltonian NH , where N is the lapse function and H is the modified super-Hamiltonian Eq. (3.27). We also use the constraint equation $H = 0$. The new momenta \tilde{P}_a and \tilde{P}_T , present in the H , are given by Eq. (3.24). In those expressions of \tilde{P}_a and \tilde{P}_T , θ and β are the parameters associated with the noncommutativity among the canonical variables and momenta, respectively (Eq. (3.17)). In the present study, we are going to consider only the contribution coming from the parameter β . In other words, we shall fix $\theta = 0$. The reason is because we do not want the resulting dynamical equations depending on terms of order greater than two in the velocities.

Assuming that $N = 1$, we obtain the following two dynamical equations for the scalar factor a ,

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{\Lambda}{3} + \frac{\rho}{3} - \frac{\beta}{3} a^{-3\alpha-2}, \quad (4.1)$$

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \Lambda - p - \beta a^{-3\alpha-2} \left(\frac{1}{3} - \alpha \right), \quad (4.2)$$

where the dot means derivative with respect to the time coordinate t , in the present gauge. The first equation is the generalization for the NC models of the Friedmann equation. Both equations reduce the corresponding dynamical equations of the commutative models when we use that $\beta = 0$.

It will be very useful to rewrite the generalized Friedmann equation (4.1) with the following form,

$$\dot{a}^2 + V(a) = 0, \quad (4.3)$$

where

$$V(a) = k - \frac{1}{3}\Lambda a^2 + \frac{\beta}{3}a^{-3\alpha} - \frac{C}{3}a^{-3\alpha-1}, \quad (4.4)$$

where C is a positive integration constant which is related to the initial fluid energy density (ρ_0). We notice that the total energy of this conservative system is equal to zero. From the observation of the potential curve $V(a)$, we shall be able to derive the qualitative dynamical behavior of $a(t)$.

It is important to notice that the NC models described by the two dynamical equations above satisfy the energy conservation equation. In order to show this result, we use both Eqs. (4.1) and (4.2) to obtain,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0. \quad (4.5)$$

This is the energy conservation equation to the commutative version of the models. Therefore, the noncommutativity does not violate the energy conservation law.

Both equations (4.1) and (4.2) are not independent. On the other hand, it can be shown that the set of solutions of Eq. (4.2) is not the same set of solutions of Eq. (4.1). However, the set of solutions of Eq. (4.2) is the same set of solutions of

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{\beta}{3}a^{-3\alpha-2} + f(t), \quad (4.6)$$

where $f(t) = f_0 a^{-3}(t)$ and f_0 is an integration constant. We may impose that the solutions of Eq. (4.2) be, also, solutions of Eq. (4.1). It can be accomplished by fixing the following initial condition on the velocity $\dot{a}(t = t_0) \equiv \dot{a}_0$,

$$\dot{a}_0 = \mp \left\{ \left(-\frac{k}{a^2} + \frac{\Lambda}{3} + \frac{8\pi G\rho}{3} - \frac{\beta}{3}a_0^{-3\alpha-2} \right) a_0^2 \right\}^{\frac{1}{2}}, \quad (4.7)$$

where $a(t = t_0) \equiv a_0$. The only way this initial condition satisfy Eq. (4.6) is when $f_0 = 0$. Since this result must be valid for all times this initial condition guarantees that all solutions $a(t)$ of Eq. (4.2) are also solutions of Eq. (4.1). In the following analysis we shall restrict our attention to the positive sign in Eq. (4.7).

In order to derive the scalar factor as a function of t , we shall, initially, observe the potential curve $V(a)$ from Eq. (4.1) and then solve Eq. (4.2). Unfortunately, for generic values of the different parameters present in Eq. (4.2), this equation does not have algebraic solutions. Therefore, we shall solve it numerically using a Fehlberg fourth-fifth order Runge-Kutta method with degree four interpolant, for each different values of the parameters.

We have four parameters in Eq. (4.2). The first one is k , which is associated with the curvature of the spatial sections and we may assume three different values: -1, 0, +1. The second one is Λ , the cosmological constant, which can be positive, negative or zero. The third one is β , the NC parameter, which can be also positive, negative or zero. It is important to mention that this last case $\beta = 0$ means that the model is commutative.

Finally, we have the parameter α , which is present in the equation of state for the perfect fluid ($p = \alpha\rho$). Each value of α defines a different perfect fluid. Here, we shall consider six different values of $\alpha = (1, 1/3, 0, -1/3, -2/3, -1)$, which represents respectively: stiff matter, radiation, dust, cosmic strings, domain walls and dark energy. Taking into account all possible values of the parameters, we have considered 162 qualitatively different cases and we solved Eq. (4.2) for all of them. In fact, we have solved Eq. (4.2) for a number of cases greater than 162 because we have considered not only the different signs for the cases of Λ and β but also for different absolute values of these parameters.

After solving Eq. (4.2) for all possible cases, mentioned above, we have reached the following general conclusions. If the parameter β is positive, it has the net effect of producing a negative acceleration in the scalar factor evolution. In cases where the universe is expanding, the presence of a positive β , will slow the expansion or even stops it and forces the universe to contract. If the parameter β is negative, it has the net effect of producing a positive acceleration in the scalar factor evolution. In cases where the universe is expanding, the presence of a negative β , will increase the expansion speed. On the other hand, in cases where the Universe is contracting, the presence of a negative β may force the Universe to expand. From these observations, we notice that β modifies the acceleration of the scale factor in the opposite way that the cosmological constant does. Next, we shall present some particular cases where the above conclusions can be clearly verified.

4.1 Universe with $k = 0$, $\alpha = 0$ and $\Lambda > 0$

For this case the Universe has flat spatial sections, it is filled with dust and the cosmological constant is positive. The potential $V(a)$ in Eq. (4.4) is given by,

$$V(a) = -\frac{\Lambda a^2}{3} - \frac{C}{3a} + \frac{\beta}{3}. \quad (4.8)$$

From the potential expression it is easy to see that this universe starts to expand from a singularity at $a = 0$. For,

$$\beta < (27C^2\Lambda/4)^{(1/3)}, \quad (4.9)$$

the Universe expands initially in a decelerated rate and later on in an accelerated rate toward $a \rightarrow \infty$. Since, in the present case $C, \Lambda > 0$, the above condition is satisfied for all cases where $\beta \leq 0$. The condition Eq. (4.9) guarantees that the potential $V(a)$ Eq. (4.8) is always negative. For $\beta = 0$, we have the commutative solution. If β is positive and satisfies Eq. (4.9), the universe expands in a smaller rate than in the commutative case. On the other hand, if β is negative the universe expands in a greater rate than in the commutative case. As an example we show in figure 1 the potential $V(a)$, Eq. (4.8). For the case where $\Lambda = 0.01$, $C = 0.01$ and $\beta = 0.01(\text{red}), 0(\text{blue}), -0.01(\text{green})$. Then, in figure 2, we show the solutions to Eq. (4.2) for the present case with the same values of the parameters of figure 1.

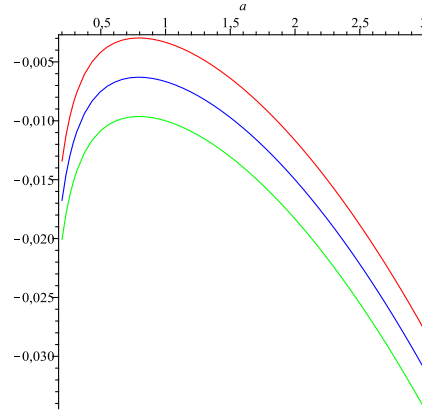


Figure 1. $V(a)$ Eq. (4.8), for $C = 0.01$, $\Lambda = 0.01$ and $\beta = 0.01$ (red), 0 (blue), -0.01 (green).

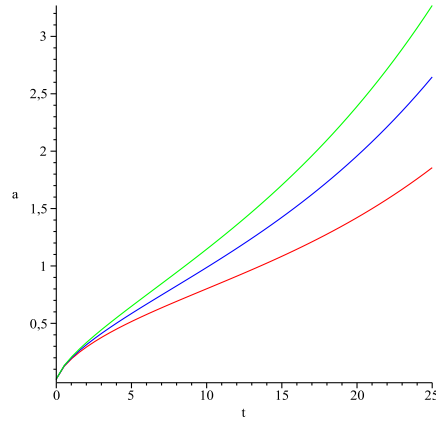


Figure 2. Solutions $a(t)$ to Eq. (4.2), for $C = 0.01$, $k = 0$, $\Lambda = 0.01$, $\alpha = 0$ and $\beta = 0.01$ (red), 0 (blue), -0.01 (green).

4.2 Universe with $k = 0$, $\alpha = 0$ and $\Lambda < 0$

For this case the universe has flat spatial sections. It is filled with dust and the cosmological constant is negative. The potential $V(a)$ Eq. (4.4) is given by,

$$V(a) = \frac{|\Lambda|a^2}{3} - \frac{C}{3a} + \frac{\beta}{3}, \quad (4.10)$$

where $|\Lambda|$ is the cosmological constant absolute value.

From the expression for the potential (4.10) it is easy to see that this universe starts to expand from a singularity at $a = 0$. Then, for all possible values of β the scale factor reaches a maximum value and starts to contract toward a *big crunch* singularity at $a = 0$. For $\beta = 0$, we have the commutative solution. If β is positive the scale factor is forced to contract in a stronger way than in the commutative case. Here, the maximum value of a is smaller than in the commutative case. On the other hand, if β is negative the scale factor will contract in a weaker way than in the commutative case. Here, the maximum value of a is greater than in the commutative case. As an example we show in figure 3 the potential $V(a)$, Eq. (4.10), for the case where $\Lambda = -0.01$, $C = 0.01$ and $\beta = 0.01$ (red), 0 (blue), -0.01 (green). Then, in figure 4, we show the solutions to Eq. (4.2) for the present case with the same values of the parameters of figure 3.

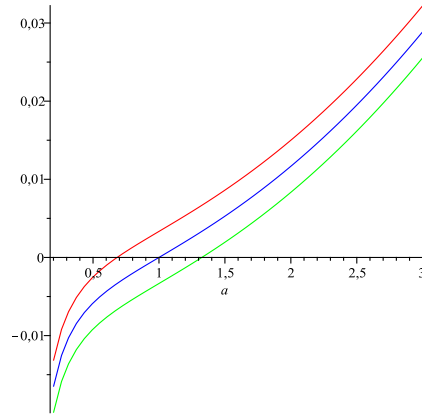


Figure 3. $V(a)$ Eq. (4.10), for $C = 0.01$, $\Lambda = -0.01$ and $\beta = 0.01$ (red), 0 (blue), -0.01 (green).

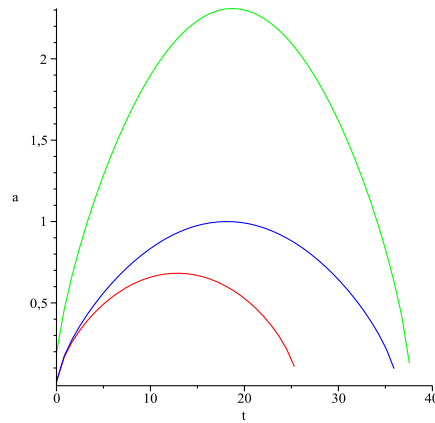


Figure 4. Solutions $a(t)$ to Eq. (4.2), for $C = 0.01$, $k = 0$, $\Lambda = -0.01$, $\alpha = 0$ and $\beta = 0.01$ (red), 0 (blue), -0.01 (green).

4.3 Universe with $k = 0$, $\alpha = 1/3$ and $\Lambda = 0$

For this case the Universe has flat spatial sections, it is filled with radiation and there is no cosmological constant. The potential $V(a)$ Eq. (4.4) is given by,

$$V(a) = -\frac{C}{3a^2} + \frac{\beta}{3a}. \quad (4.11)$$

Here, we have a very interesting situation. For $\beta \leq 0$ the universe starts to expand from a singularity at $a = 0$. It expands in a decelerated rate until it reaches asymptotically $a \rightarrow \infty$. For $\beta = 0$, we have the commutative solution. This commutative case is well-known in the literature and it corresponds to Friedmann equation Eq.(4.3) that can be solved to give the algebraic solution: $a(t) = At^{(1/2)}$, where A is a constant. If β is negative the universe expands in a rate greater than the commutative case. On the other hand, if β is positive the universe expands up to a maximum size given by c/β , then it is forced to contract toward a *big crunch* singularity at $a = 0$. Therefore, we see that the NC parameter may change drastically the universe evolution. As an example we show in figure 5 the potential $V(a)$, Eq. (4.11), for the case where $\Lambda = -0.01$, $C = 0.01$ and $\beta = 0.01$ (red), 0 (blue), -0.01 (green). Then, in figure 6, we show the solutions to Eq. (4.2) for the present case with the same values of the parameters of figure 5. It is important to mention that the term depending on β in Eq. (4.2) is not present in this case. The solutions of Eq. (4.2) depend on β , in the present case, due to the initial condition Eq. (4.7).

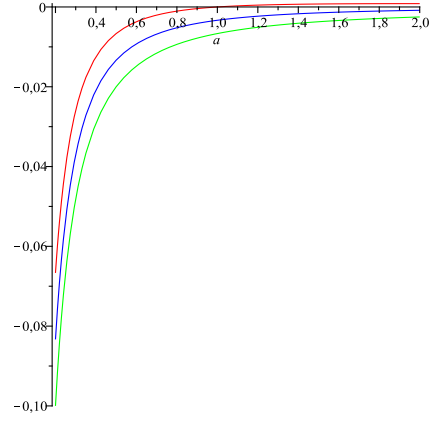


Figure 5. $V(a)$ Eq. (4.11), for $C = 0.01$ and $\beta = 0.01$ (red), 0 (blue), -0.01 (green).

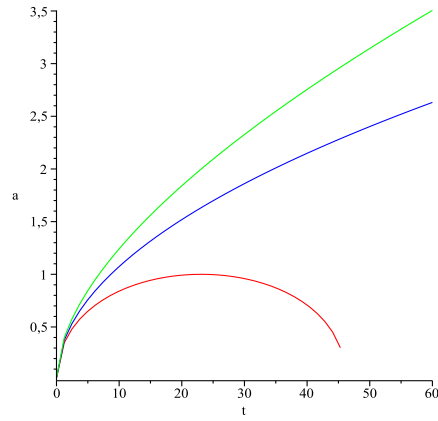


Figure 6. Solutions $a(t)$ to Eq. (4.2), for $C = 0.01$, $k = 0$, $\Lambda = 0$, $\alpha = 1/3$ and $\beta = 0.01$ (red), 0 (blue), -0.01 (green).

4.4 Universe with $k = -1$, $\alpha = -2/3$ and $\Lambda < 0$

For this case the Universe has constant negatively curved spatial sections, it is filled with a domain wall perfect fluid and there is a negative cosmological constant. The potential $V(a)$ in Eq. (4.4) is given by,

$$V(a) = -1 - \frac{\Lambda}{3}a^2 + \frac{\beta}{3}a^2 - \frac{C}{3}a. \quad (4.12)$$

Here, we have the situation opposite to the previous case. For $\beta > \Lambda$, the universe starts to expand from $a = 0$, which is not a singularity. Then, the scale factor reaches a maximum value,

$$a_{max} = (C + \sqrt{C^2 + 12(\beta - \Lambda)})/[2(\beta - \Lambda)],$$

and starts to contract toward $a = 0$.

Since $\Lambda < 0$, the Universe is bounded for the commutative solution. If β is positive the scale factor is forced to contract in a stronger way than in the commutative case. Here, the maximum value of a is smaller than in the commutative case.

On the other hand, if $\beta \leq \Lambda$ the Universe starts to expand from $a = 0$ and accelerates its expansion toward $a \rightarrow \infty$. Therefore, we will see another example where the NC parameter may change drastically the universe evolution. As an example we show in figure 7 the potential $V(a)$, Eq. (4.12), for the case where $\Lambda = -0.01$, $C = 0.01$ and $\beta = 0.011$ (red), 0 (blue), -0.011 (green). Here, we can find an algebraic solution for the scalar factor as a function of time. Using Eqs. (4.1) and (4.2), and using the values of the parameters for the present case, we obtain the following differential equation,

$$\ddot{a} + \omega^2 a = \frac{C}{6}. \quad (4.13)$$

The general solution of this equation is given by,

$$a(t) = Ae^{i\omega t} + Be^{-i\omega t} + \frac{C}{6\omega^2}. \quad (4.14)$$

where A, B are integration constants and $\omega^2 = (\beta - \Lambda)/3$.

If $\omega^2 > 0$ Eq. (4.13) represents a driven harmonic oscillator with a constant driven force. The solution, Eq. (4.14), is oscillatory. On the other hand, if $\omega^2 < 0$ Eq. (4.13) represents an unbounded system and the solution Eq. (4.14) grows exponentially.

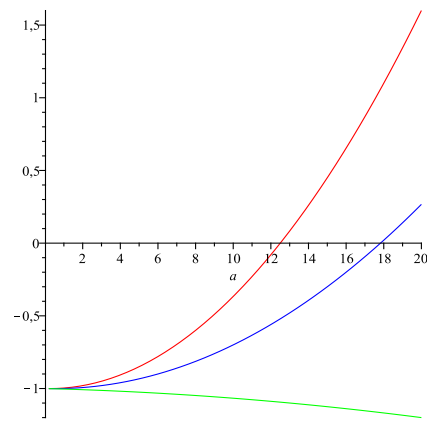


Figure 7. $V(a)$ Eq. (4.12), for $\Lambda = -0.01$, $C = 0.01$ and $\beta = 0.01$ (red), 0 (blue), -0.01 (green).

5 The noncommutativity and the Universe evolution

Taking into account the above results we can see that the presence of β in the cosmological equations may modify in a fundamental way the evolution of the Universe. In order to gain a better insight about the role of β , let us rewrite the generalized Friedmann equation (4.1) in the following way,

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G\rho}{3} + \tilde{\Lambda}(a), \quad (5.1)$$

where

$$\tilde{\Lambda}(a) = (1/3)(\Lambda - \beta a^{-3\alpha-2}).$$

We may interpret $\tilde{\Lambda}$ as a function of the scale factor that generalizes the cosmological constant. It is important to notice that, the contributions coming from Λ and the term depending on β oppose to each other. In order for them to reinforce each other, Λ and β must have opposite signs.

Depending on the value of α , we may have two entirely different scenarios for the evolution of the Universe described by $\tilde{\Lambda}$.

5.1 First scenario

For $\alpha > -2/3$, the modulus of the term $\beta a^{-3\alpha-2}$ decreases when a increases. This behavior agrees with the idea that noncommutativity should be most important at the beginning of the Universe. After some time, when the universe has evolved considerably a is large enough so that $\tilde{\Lambda} \approx \Lambda/3$. From this moment the Universe evolves without any remembrance of a NC phase.

5.2 Second scenario

For $\alpha < -2/3$, the modulus of the term $\beta a^{-3\alpha-2}$ increases when a increases. After some time, when the Universe has evolved considerably a is large enough so that $\tilde{\Lambda} \approx -\beta a^{-3\alpha-2}/3$. From this moment the Universe evolution is dominated by the NC term.

If $\beta > 0$, the Universe will eventually reach a maximum size and contract again toward $a \rightarrow 0$. On the other hand, if $\beta < 0$, the Universe will expand forever in an accelerated rate. Here, we have a very interesting possibility. It is possible to consider the NC effects as a candidate to explain the present accelerated expansion of the Universe [43],[44]. In fact, some authors have already considered this possibility using classical, NC cosmological models different from ours [34],[35],[36].

As a matter of completeness, we observe that for $\alpha = -2/3$, $\tilde{\Lambda}$ is a constant. It is given by, $\tilde{\Lambda} = (1/3)(\Lambda - \beta)$. In this case, the NC parameter can be compared with the cosmological constant. If $\Lambda = 0$, the NC parameter plays the role of a cosmological constant with opposite sign.

6 Final considerations and conclusions

One of the mysteries that defies the theoretical physics today is how to promote the unification of two independent sets of very different concepts. Namely, one set that rules the microscopic world, i.e., the quantum mechanics and the other that govern the macroscopic Universe, i.e., general relativity. The main motivation to unify both disconnected (so far) sets is to understand the physics of the early Universe and the physics that rules the origin and the physical structure of the black holes. In few words, we are looking for the generalized idea of the so called quantum gravity.

As we know the NC parameter has its value measured at the Planck scale. Besides, the string theory, a candidate to carry out this unification, embedded in a magnetic background (to explain it in a nutshell), was found to have a NC algebra. In view of these two theoretical facts it is natural to believe that to investigate NC theories can be one of the adequate paths to conduct us to such unification.

It is the main objective of this work to pursue this target. To accomplish this, we used the generalized NC symplectic formalism to introduce naturally noncommutativity inside the equations that provide the dynamics of the Universe, i.e., the Friedmann equations. With this procedure, we introduced a Planckian object inside the classical equations of motion of the Universe.

We have constructed NC versions of several Friedmann-Robertson-Walker cosmological models. These models may have positive, negative or zero spatial sections curvatures. They are coupled to different types of perfect fluids and they may have a positive, negative or zero cosmological constant. We have used the Schutz variational formalism in order to write the proper Hamiltonian for the models. We have performed a complete analysis of the evolution of the Universes described by each model. We solved the dynamical equations for the scale factor.

Based on the results obtained here we have concluded that, if the parameter β is positive, it has the final effect of producing a negative acceleration in the scalar factor evolution. In cases where the Universe is expanding, the presence of a positive β , will slow the expansion or even stops it and forces the Universe to contract. If the parameter β is negative, it has the effect of producing a positive acceleration in the scalar factor evolution. In cases where the Universe is expanding, the presence of a negative β , will increase the expansion speed. On the other hand, in cases where the Universe is contracting, the presence of a negative β may force the Universe to expand. From these observations, we notice that β modifies the acceleration of the scale factor in the opposite way that the cosmological constant does. We have also presented some particular cases where the above conclusions could be clearly verified. Finally, based on our results, we construct a function ($\tilde{\Lambda}$) that depends on a , β and α and generalizes the cosmological constant. We have discussed the possible scenarios for the Universe evolution predicted by $\tilde{\Lambda}$, depending on the value of α .

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